QCE Mathematical Methods Trial Examination Paper 1:Tech-free Section 1

Kilbaha Education

Quality educational content

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Trial assessment 2021

Multiple choice question book

Mathematical Methods

Paper 1— Technology-free

Section 1

Instructions

- Answer all questions in the question and response book.
- This book will not be marked.

QUESTION 1

For the graph of $y = \log_e(3x-2)$ which of the following is correct?

(A) The graph crosses the *x*-axis at $x = \frac{2}{3}$ and has a range of R.

(B) The graph crosses the *x*-axis at $x = 1$ and has a maximal domain of $\left(\frac{2}{3}, \infty\right)$.

(C) The graph crosses the *y*-axis at $y = \log_e(2)$ and has a maximal domain of R.

(D) The graph does not cross the *x* or *y*-axis and has a maximal domain of $\left(\frac{2}{3}, \infty\right)$.

QUESTION 2

A certain curve has a gradient given by $4e^{-\frac{x}{2}}$, the particular curve which passes through the origin, is given by

(A) $y = -2e^{-\frac{x}{2}}$

(B) $y = -8e^{-\frac{x}{2}}$

(C) $y = 2\left(1-e^{-\frac{x}{2}}\right)$

(D) $y = 8\left(1-e^{-\frac{x}{2}}\right)$

QCE Mathematical Methods Trial Examination Paper 1:Tech-free Section 2

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School name

Given name/s

Family name

Trial assessment 2021

Question and response book

Mathematical Methods

Paper 1— Technology-free

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are not allowed.
- QCAA formula sheet provided.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Section 1

Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.

– On the additional pages, write the question number you are responding to.
– Cancel any incorrect response by ruling a single diagonal line through your work.
– Write the page number of your alternative/additional response, i.e. See page …
– If you do not do this, your original response will be marked.

- This section has nine questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

a) If $\frac{d}{dx}\left(\frac{\log_e(2x)}{2x^2}\right)=\frac{1}{ax^n}\left(1-b\log_e(2x)\right)$, find the values of a, b and n.

[2 marks]

b) If $f(x)=\sin^3(2x)$, evaluate $f'\left(\frac{\pi}{6}\right)$.

[2 marks]

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$\frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = \pi r^2 h$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}(2t_1 + (n-1)d) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_a(a)}$

Calculus	
$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c$
$\frac{d}{dx}\left(e^x\right)=e^x$	$\int e^x dx=e^x+c$
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\left(x\right)+c$
$\frac{d}{dx}\left(\sin\left(x\right)\right)=\cos\left(x\right)$	$\int \sin\left(x\right)dx=-\cos\left(x\right)+c$
$\frac{d}{dx}\left(\cos\left(x\right)\right)=-\sin\left(x\right)$	$\int \cos\left(x\right)dx=\sin\left(x\right)+c$

chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2=a^2+b^2-2ab\cos\left(C\right)$
sine rule	$\frac{a}{\sin\left(A\right)}=\frac{b}{\sin\left(B\right)}=\frac{c}{\sin\left(C\right)}$
area of a triangle	$\text{area}=\frac{1}{2}bc\sin\left(A\right)$
Pythagorean identity	$\sin^2\left(A\right)+\cos^2\left(A\right)=1$

Probability		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + ... + \binom{n}{r}x^{n-r}y^r + ... + y^n$	
binomial probability	$\Pr(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} x\, p(x)\, dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2\, p(x)\, dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
probability of independent events	$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$	
conditional probability	$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	

QCE Mathematical Methods Suggested Solutions Trial Paper 1 Tech-free

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QUESTION 1 ANSWER B

$y = \log_e(3x-2)$, The graph does not cross the *y*-axis, and crosses the *x*-axis when $y = 0$,since $\log_e(1) = 0$ $3x-2=1$ so that $x=1$ and has a maximal domain when $3x-2>0$, $x > \frac{2}{3}$ or $\left(\frac{2}{3}, \infty\right)$.

QUESTION 2 ANSWER D

$\frac{dy}{dx} = 4e^{-\frac{x}{2}} \Rightarrow y = \int 4e^{-\frac{x}{2}}\,dx = -8e^{-\frac{x}{2}} + c$

now when $x=0$ $y=0$, $0 = -8 + c \Rightarrow c = 8$, $y = 8\left(1 - e^{-\frac{x}{2}}\right)$

QUESTION 11

a) $y = \dfrac{\log_e(2x)}{2x^2}$ differentiating using the quotient rule

let $u = \log_e(2x)$ $\quad v = 2x^2$

$\dfrac{du}{dx} = \dfrac{1}{x}$ $\quad \dfrac{dv}{dx} = 4x$ M1

$$\frac{dy}{dx} = \frac{2x^2 \times \frac{1}{x} - 4x\log_e(2x)}{\left(2x^{2^2}\right)} = \frac{2x - 4x\log_e(2x)}{4x^4}$$

$$\frac{dy}{dx} = \frac{2x\left(1 - 2\log_e(x)\right)}{4x^4}$$

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\log_e(2x)}{2x^2}\right) = \frac{1}{2x^3}\left(1 - 2\log_e(2x)\right)$$ A1

$a = 2$, $n = 3$, $b = 2$

b) If $f(x) = y = \sin^3(2x) = u^3$ where $u = \sin(2x)$,using the chain rule

$\dfrac{dy}{du} = 3u^2$ $\quad \dfrac{du}{dx} = 2\cos(2x)$

$$f'(x) = \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx} = 6\sin^2(2x)\cos(2x)$$ M1

$$f'\left(\frac{\pi}{6}\right) = 6\sin^2\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) = 6 \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{1}{2}$$

$$f'\left(\frac{\pi}{6}\right) = \frac{9}{4}$$ A1