

QCE
Mathematical
Methods
Trial Examination
Paper 1: Tech-free
Section 1



Kilbaha Education

Quality educational content

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Trial assessment 2021

Multiple choice question book

Mathematical Methods

Paper 1— Technology-free

Section 1

Instructions

- Answer all questions in the question and response book.
 - This book will not be marked.
-

QUESTION 1

For the graph of $y = \log_e(3x - 2)$ which of the following is correct?

- (A) The graph crosses the x -axis at $x = \frac{2}{3}$ and has a range of R .
- (B) The graph crosses the x -axis at $x = 1$ and has a maximal domain of $\left(\frac{2}{3}, \infty\right)$.
- (C) The graph crosses the y -axis at $y = \log_e(2)$ and has a maximal domain of R .
- (D) The graph does not cross the x or y -axis and has a maximal domain of $\left(\frac{2}{3}, \infty\right)$.

QUESTION 2

A certain curve has a gradient given by $4e^{-\frac{x}{2}}$, the particular curve which passes through the origin, is given by

- (A) $y = -2e^{-\frac{x}{2}}$
- (B) $y = -8e^{-\frac{x}{2}}$
- (C) $y = 2\left(1 - e^{-\frac{x}{2}}\right)$
- (D) $y = 8\left(1 - e^{-\frac{x}{2}}\right)$

QCE
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Section 2



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School name

Given name/s

Family name

Trial assessment 2021

Question and response book

Mathematical Methods

Paper 1— Technology-free

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response book.
- Calculators are not allowed.
- QCAA formula sheet provided.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Section 1

Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

	A	B	C	D
1.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Section 2

Instructions

- Write using black or blue pen.
 - Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
 - If you need more space for a response, use the additional pages at the back of this booklet.
 - On the additional pages, write the question number you are responding to.
 - Cancel any incorrect response by ruling a single diagonal line through your work.
 - Write the page number of your alternative/additional response, i.e. See page ...
 - If you do not do this, your original response will be marked.
 - This section has nine questions and is worth 50 marks.
-

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

a) If $\frac{d}{dx}\left(\frac{\log_e(2x)}{2x^2}\right) = \frac{1}{ax^n}(1 - b\log_e(2x))$, find the values of a , b and n .

[2 marks]

b) If $f(x) = \sin^3(2x)$, evaluate $f'\left(\frac{\pi}{6}\right)$.

[2 marks]

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$\frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = \pi r^2 h$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}(2t_1 + (n-1)d) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r - 1)}$ $S_\infty = \frac{t_1}{(1 - r)}, r < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n \log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c$	
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + c$	
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c$	
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$	
$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$	
chain rule	If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2 = a^2 + b^2 - 2ab\cos(C)$
sine rule	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
area of a triangle	$\text{area} = \frac{1}{2}bc \sin(A)$
Pythagorean identity	$\sin^2(A) + \cos^2(A) = 1$

Probability		
binomial theorem	$(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{r}x^{n-r}y^r + \dots + y^n$	
binomial probability	$\Pr(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable X	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$\text{Var}(X) = \sum p_i (x_i - \mu)^2$
continuous random variable X	mean	$E(X) = \mu = \int_{-\infty}^{\infty} x p(x) dx$
	variance	$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for p	$\left(\hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$	
general addition rule for probability	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
probability of independent events	$P(A \cap B) = P(A) \times P(B)$	
conditional probability	$P(A B) = \frac{P(A \cap B)}{P(B)}$	

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QUESTION 1 ANSWER B

$y = \log_e(3x-2)$, The graph does not cross the y -axis, and crosses the x -axis when $y = 0$, since $\log_e(1) = 0$ $3x-2=1$ so that $x=1$ and has a maximal domain when $3x-2 > 0$, $x > \frac{2}{3}$ or $(\frac{2}{3}, \infty)$.

QUESTION 2 ANSWER D

$$\frac{dy}{dx} = 4e^{-\frac{x}{2}} \Rightarrow y = \int 4e^{-\frac{x}{2}} dx = -8e^{-\frac{x}{2}} + c$$

now when $x=0$ $y=0$, $0 = -8 + c \Rightarrow c = 8$, $y = 8\left(1 - e^{-\frac{x}{2}}\right)$

QUESTION 3 ANSWER C

$g(x) = y = e^{\sqrt{x}} = e^u$ $u = \sqrt{x} = x^{\frac{1}{2}}$ using the chain rule

$$\frac{dy}{du} = e^u \quad \frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$$

$$\frac{dy}{dx} = g'(x) = \frac{dy}{du} \frac{du}{dx} = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

$$g'(4) = \frac{e^{\sqrt{4}}}{2\sqrt{4}} = \frac{e^2}{4}$$

QUESTION 4 ANSWER B

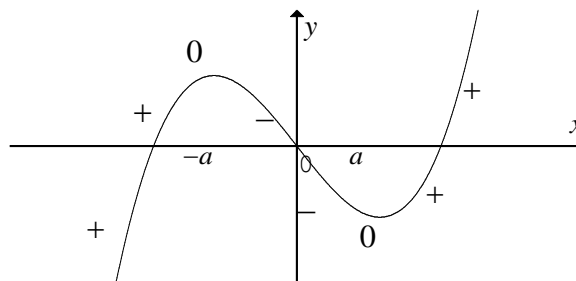
$$f'(x) < 0 \text{ for } -a < x < a$$

$$f'(x) > 0 \text{ for } x > a \text{ or } x < -a$$

$$f'(-a) = 0 \text{ and } f'(a) = 0$$

the signs of the gradient are shown.

The graph of f has stationary points at both $x = \pm a$. A local maximum at $x = -a$ and a local minimum at $x = a$.



QUESTION 5 ANSWER C

Three right rectangles, each of width $h = \frac{\pi}{6}$

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$y = \sin(x)$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1

The shaded area of the three rectangles is

$$A = \frac{\pi}{6} \left(\frac{1}{2} + \frac{\sqrt{3}}{2} + 1 \right) = \frac{\pi}{6} \left(\frac{3 + \sqrt{3}}{2} \right) = \frac{\pi(3 + \sqrt{3})}{12}$$

QUESTION 11

a) $y = \frac{\log_e(2x)}{2x^2}$ differentiating using the quotient rule

let $u = \log_e(2x)$ $v = 2x^2$

$\frac{du}{dx} = \frac{1}{x}$ $\frac{dv}{dx} = 4x$ M1

$\frac{dy}{dx} = \frac{2x^2 \times \frac{1}{x} - 4x \log_e(2x)}{(2x^2)^2} = \frac{2x - 4x \log_e(2x)}{4x^4}$

$\frac{dy}{dx} = \frac{2x(1 - 2\log_e(x))}{4x^4}$

$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{\log_e(2x)}{2x^2} \right) = \frac{1}{2x^3} (1 - 2\log_e(2x))$ A1

$a = 2, n = 3, b = 2$

b) If $f(x) = y = \sin^3(2x) = u^3$ where $u = \sin(2x)$, using the chain rule

$\frac{dy}{du} = 3u^2$ $\frac{du}{dx} = 2\cos(2x)$

$f'(x) = \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 6\sin^2(2x)\cos(2x)$ M1

$f'\left(\frac{\pi}{6}\right) = 6\sin^2\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{3}\right) = 6 \times \left(\frac{\sqrt{3}}{2}\right)^2 \times \frac{1}{2}$

$f'\left(\frac{\pi}{6}\right) = \frac{9}{4}$ A1

QUESTION 12

a) $\log_8(x+5) + \log_8(3x-1) = 2$

$\log_8(x+5)(3x-1) = 2$ M1

$(x+5)(3x-1) = 8^2 = 64$

$3x^2 + 14x - 5 = 64$

$3x^2 + 14x - 69 = 0$

$(3x+23)(x-3) = 0$ A1

$x = -\frac{23}{3}, 3$ but $x > \frac{1}{3}$

$x = 3$ as the only answer A1