QCE Mathematical Methods Trial Examination Paper 2:Tech-active Section 1

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Trial assessment 2021

Multiple choice question book

Mathematical Methods

Paper 2— Technology-active

Section 1

Instructions

- Answer all questions in the question and response book.
- This book will not be marked.

QUESTION 1

If $\int_0^b \frac{x}{x^2+4}dx = \log_e\left(\sqrt{2}\right)$ then

A) $b = 2$

B) $b = \sqrt{2}$

C) $b = 2\sqrt{\sqrt{2}-1}$

D) $b = 1$

QUESTION 2

A discrete random variable has a binomial distribution. The expression $1-\left(0.65^8+8(0.35)(0.65)^7\right)$ represents the probability of

A) more than one success in eight trials each with probability of success equal to 0.65.

B) more than one success in eight trials each with probability of success equal to 0.35.

C) at least one success in eight trials each with probability of success equal to 0.65.

D) at least one success in eight trials each with probability of success equal to 0.35.

QCE Mathematical Methods Trial Examination Paper 2:Tech-active Section 2

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School name

Given name/s

Family name

Trial assessment 2021

Question and response book

Mathematical Methods

Paper 2— Technology-active

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this questions and response book.
- QCAA-approved calculator permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (50 marks)

- 9 short response questions

Section 1

Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.

– On the additional pages, write the question number you are responding to.
– Cancel any incorrect response by ruling a single diagonal line through your work.
– Write the page number of your alternative/additional response, i.e. See page …
– If you do not do this, your original response will be marked.

- This section has nine questions and is worth 50 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

The area bounded by the graph of $y = 4\sin\left(\frac{2\pi x}{3}\right)$, the x-axis, the origin, and the line $x = b$ is equal to $\frac{3}{\pi}$. Determine the smallest possible value of b, where $b > 0$.

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$\frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = \pi r^2 h$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}(2t_1 + (n-1)d) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r-1)}$ $S_\infty = \frac{t_1}{(1-r)}, \ \lvert r \rvert < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n\log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_a(a)}$

Calculus	
$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c$
$\frac{d}{dx}\left(e^x\right)=e^x$	$\int e^x dx=e^x+c$
$\frac{d}{dx}\left(\log_e\left(x\right)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\left(x\right)+c$
$\frac{d}{dx}\left(\sin\left(x\right)\right)=\cos\left(x\right)$	$\int \sin\left(x\right)dx=-\cos\left(x\right)+c$
$\frac{d}{dx}\left(\cos\left(x\right)\right)=-\sin\left(x\right)$	$\int \cos\left(x\right)dx=\sin\left(x\right)+c$

chain rule	If $h(x)=f\left(g(x)\right)$ then $h'(x)=f'\left(g(x)\right)g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{\left(g(x)\right)^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$

Trigonometry	
cosine rule	$c^2=a^2+b^2-2ab\cos\left(C\right)$
sine rule	$\frac{a}{\sin\left(A\right)}=\frac{b}{\sin\left(B\right)}=\frac{c}{\sin\left(C\right)}$
area of a triangle	$\text{area}=\frac{1}{2}bc\sin\left(A\right)$
Pythagorean identity	$\sin^2\left(A\right)+\cos^2\left(A\right)=1$

Probability		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1}x^{n-1}y + ... + \binom{n}{r}x^{n-r}y^r + ... + y^n$	
binomial probability	$\Pr(X=r) = \binom{n}{r}p^r(1-p)^{n-r}$	
discrete random variable *X*	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$Var(X) = \sum p_i (x_i - \mu)^2$
continuous random variable *X*	mean	$E(X) = \mu = \int_{-\infty}^{\infty} x\, p(x)\, dx$
	variance	$Var(X) = \int_{-\infty}^{\infty} (x-\mu)^2\, p(x)\, dx$
binomial distribution	mean	np
	variance	$np(1-p)$
sample proportion	mean	p
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
approximate confidence interval for *p*	$\left(\hat{p} - z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$	
general addition rule for probability	$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$	
probability of independent events	$\Pr(A \cap B) = \Pr(A) \times \Pr(B)$	
conditional probability	$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$	

QCE Mathematical Methods Suggested Solutions Trial Paper 2 Tech-active

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QUESTION 1 ANSWER A

$$\int_0^b \frac{x}{x^2+4}\,dx = \left[\frac{1}{2}\log_e\left(x^2+4\right)\right]_0^b$$

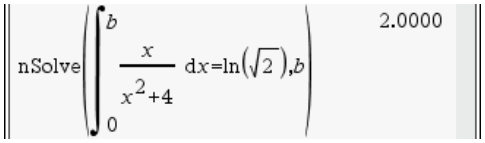
$$= \frac{1}{2}\log_e\left(b^2+4\right) - \frac{1}{2}\log_e(4) = \frac{1}{2}\log_e\left(\frac{b^2+4}{4}\right) = \log_e\left(\sqrt{2}\right) = \frac{1}{2}\log_e(2)$$

$$\frac{b^2+4}{4} = 2$$

$$b^2+4 = 8$$

$$b^2 = 4$$

$b = 2$, $b > 0$



QUESTION 2 ANSWER B

$X \overset{d}{=} \text{Bi}(n = ?, p = ?)$

$$\Pr(\text{more than one}) = \Pr(X > 1) = 1 - \left[\Pr(X = 0) + \Pr(X = 1)\right] = 1 - \left(0.65^8 + 8(0.35)(0.65)^7\right)$$

Now $\Pr(X = 0) = q^n$ and $\Pr(X = 1) = npq^{n-1}$

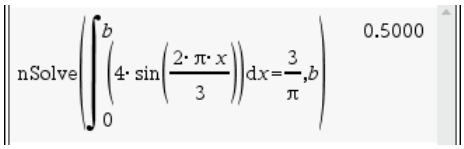
$n = 8$, $q = 0.65$ and $p = 0.35$

8 trials and $p = \Pr(\text{success}) = 0.35$

QUESTION 11

$$\int_0^b 4\sin\left(\frac{2\pi x}{3}\right)=\frac{3}{\pi}$$ A1

$$\left[-\frac{6}{\pi}\cos\left(\frac{2\pi x}{3}\right)\right]_0^b=\frac{3}{\pi}$$



A1

$$\cos\left(\frac{2\pi b}{3}\right)-1=-\frac{1}{2}$$

$$\cos\left(\frac{2\pi b}{3}\right)=\frac{1}{2}$$ M1

$$\frac{2\pi b}{3}=\frac{\pi}{3}$$

$$b=\frac{1}{2}$$ A1