

**QCE**  
**Mathematical**  
**Methods**  
**Trial Examination**  
**Paper 2: Tech-active**  
**Section 1**



**Kilbaha Education**

Quality educational content

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**Trial assessment 2021**

Multiple choice question book

# Mathematical Methods

Paper 2— Technology-active

# Section 1

## Instructions

- Answer all questions in the question and response book.
  - This book will not be marked.
- 

### QUESTION 1

If  $\int_0^b \frac{x}{x^2+4} dx = \log_e(\sqrt{2})$  then

- A)  $b = 2$
- B)  $b = \sqrt{2}$
- C)  $b = 2\sqrt{\sqrt{2}-1}$
- D)  $b = 1$

### QUESTION 2

A discrete random variable has a binomial distribution. The expression  $1 - (0.65^8 + 8(0.35)(0.65)^7)$  represents the probability of

- A) more than one success in eight trials each with probability of success equal to 0.65.
- B) more than one success in eight trials each with probability of success equal to 0.35.
- C) at least one success in eight trials each with probability of success equal to 0.65.
- D) at least one success in eight trials each with probability of success equal to 0.35.

### QUESTION 3

The amount of bacteria grows at a rate of  $5e^{0.2t}$  grams per day, where  $t \geq 0$ , is the time measured in days. Over the time interval from  $t = 0$  to  $t = 10$  days, the amount in grams has grown by

- A) 159.73
- B) 15.97
- C) 6.39
- D) 3.19

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**Section 2**



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**Trial assessment 2021**

Question and response book

# Mathematical Methods

## Paper 2— Technology-active

**Time allowed**

- Perusal time – 5 minutes
- Working time – 90 minutes

**General instructions**

- Answer all questions in this questions and response book.
- QCAA-approved calculator permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

**Section 1 ( 10 marks )**

- 10 multiple choice questions

**Section 2 ( 50 marks )**

- 9 short response questions

## Section 1

### Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
Example:	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

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	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
<b>1.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>2.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>3.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>4.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>5.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>6.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>7.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>8.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>9.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
<b>10.</b>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

## Section 2

### Instructions

- Write using black or blue pen.
  - Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
  - If you need more space for a response, use the additional pages at the back of this booklet.
    - On the additional pages, write the question number you are responding to.
    - Cancel any incorrect response by ruling a single diagonal line through your work.
    - Write the page number of your alternative/additional response, i.e. See page ...
    - If you do not do this, your original response will be marked.
  - This section has nine questions and is worth 50 marks.
- 

**DO NOT WRITE ON THIS PAGE**

**THIS PAGE WILL NOT BE MARKED**



**QUESTION 11** (4 marks)

The area bounded by the graph of  $y = 4 \sin\left(\frac{2\pi x}{3}\right)$ , the  $x$ -axis, the origin, and the line  $x = b$  is equal to  $\frac{3}{\pi}$ . Determine the smallest possible value of  $b$ , where  $b > 0$ .

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Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$\frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = \pi r^2 h$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Sequences and series	
arithmetic sequence	$t_n = t_1 + (n-1)d$ $S_n = \frac{n}{2}(2t_1 + (n-1)d) = \frac{n}{2}(t_1 + t_n)$
geometric sequence	$t_n = t_1 r^{(n-1)}$ $S_n = t_1 \frac{(r^n - 1)}{(r - 1)}$ $S_\infty = \frac{t_1}{(1 - r)},  r  < 1$

Logarithms	
exponents and logarithms	$a^x = b \Leftrightarrow x = \log_a(b)$
logarithmic laws	$\log_a(x) + \log_a(y) = \log_a(xy)$ $\log_a(x) - \log_a(y) = \log_a\left(\frac{x}{y}\right)$ $\log_a(x^n) = n \log_a(x)$ $\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$

Calculus		
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1} x^{n+1} + c$	
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + c$	
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c$	
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$	
$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$	
<b>chain rule</b>	If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
<b>product rule</b>	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
<b>quotient rule</b>	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Trigonometry	
<b>cosine rule</b>	$c^2 = a^2 + b^2 - 2ab\cos(C)$
<b>sine rule</b>	$\frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$
<b>area of a triangle</b>	$\text{area} = \frac{1}{2}bc \sin(A)$
<b>Pythagorean identity</b>	$\sin^2(A) + \cos^2(A) = 1$

Probability		
<b>binomial theorem</b>	$(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{r}x^{n-r}y^r + \dots + y^n$	
<b>binomial probability</b>	$\Pr(X = r) = \binom{n}{r}p^r(1-p)^{n-r}$	
<b>discrete random variable <math>X</math></b>	mean	$E(X) = \mu = \sum p_i x_i$
	variance	$\text{Var}(X) = \sum p_i (x_i - \mu)^2$
<b>continuous random variable <math>X</math></b>	mean	$E(X) = \mu = \int_{-\infty}^{\infty} x p(x) dx$
	variance	$\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x) dx$
<b>binomial distribution</b>	mean	$np$
	variance	$np(1-p)$
<b>sample proportion</b>	mean	$p$
	standard deviation	$\sqrt{\frac{p(1-p)}{n}}$
<b>approximate confidence interval for <math>p</math></b>	$\left( \hat{p} - z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \right)$	
<b>general addition rule for probability</b>	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$	
<b>probability of independent events</b>	$P(A \cap B) = P(A) \times P(B)$	
<b>conditional probability</b>	$P(A   B) = \frac{P(A \cap B)}{P(B)}$	

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**QUESTION 1 ANSWER A**

$$\int_0^b \frac{x}{x^2+4} dx = \left[ \frac{1}{2} \log_e(x^2+4) \right]_0^b$$

$$= \frac{1}{2} \log_e(b^2+4) - \frac{1}{2} \log_e(4) = \frac{1}{2} \log_e\left(\frac{b^2+4}{4}\right) = \log_e(\sqrt{2}) = \frac{1}{2} \log_e(2)$$

$$\frac{b^2+4}{4} = 2$$

$$b^2+4 = 8$$

$$b^2 = 4$$

$$b = 2, b > 0$$

n.Solve  $\left( \int_0^b \frac{x}{x^2+4} dx = \ln(\sqrt{2}), b \right)$  2.0000

**QUESTION 2 ANSWER B**

$$X \stackrel{d}{=} \text{Bi}(n=?, p=?)$$

$$\Pr(\text{more than one}) = \Pr(X > 1) = 1 - [\Pr(X = 0) + \Pr(X = 1)] = 1 - (0.65^8 + 8(0.35)(0.65)^7)$$

$$\text{Now } \Pr(X = 0) = q^n \text{ and } \Pr(X = 1) = npq^{n-1}$$

$$n = 8, q = 0.65 \text{ and } p = 0.35$$

$$8 \text{ trials and } p = \Pr(\text{success}) = 0.35$$

**QUESTION 3 ANSWER A**

$$\frac{dN}{dt} = 5e^{0.2t} = 5e^{\frac{t}{5}}$$

$$N = \int_0^{10} 5e^{\frac{t}{5}} dt = \left[ 25e^{\frac{t}{5}} \right]_0^{10}$$

$$= 25 \left( e^{\frac{10}{5}} - e^0 \right) = 25(e^2 - 1) = 159.73$$

$\int_0^{10} (5 \cdot e^{0.2 \cdot t}) dt$  159.726

**QUESTION 4 ANSWER D**

$$C(t) = 15 - 5 \cos\left(\frac{2\pi}{365}(t+10)\right)$$

$$\frac{dC}{dt} = \frac{5 \times 2\pi}{365} \sin\left(\frac{2\pi}{365}(t+10)\right)$$

$$\left. \frac{dC}{dt} \right|_{t=90} = \frac{10\pi}{365} \sin\left(\frac{200\pi}{365}\right) \approx 0.085$$

$\left. \frac{d}{dt} \left( 15 - 5 \cdot \cos\left(\frac{2 \cdot \pi}{365} \cdot (t+10)\right) \right) \right|_{t=90}$  0.085097

**QUESTION 11**

$$\int_0^b 4 \sin\left(\frac{2\pi x}{3}\right) dx = \frac{3}{\pi} \quad \text{A1}$$

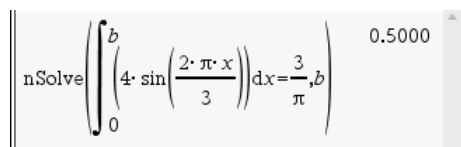
$$\left[-\frac{6}{\pi} \cos\left(\frac{2\pi x}{3}\right)\right]_0^b = \frac{3}{\pi} \quad \text{A1}$$

$$\cos\left(\frac{2\pi b}{3}\right) - 1 = -\frac{1}{2} \quad \text{A1}$$

$$\cos\left(\frac{2\pi b}{3}\right) = \frac{1}{2} \quad \text{M1}$$

$$\frac{2\pi b}{3} = \frac{\pi}{3}$$

$$b = \frac{1}{2} \quad \text{A1}$$



**QUESTION 12**

a) crosses the  $x$ -axis when  $x^3 - 4e^{-2x} = 0$ , solving gives  $x = 0.88$ ,  $(0.88, 0)$  A1

b)  $f'(x) = 3x^2 + 8e^{-2x}$   
 $f''(x) = 6x - 16e^{-2x}$ , for inflexion points  $f''(x) = 0$ , solving M1  
 $3x - 8e^{-2x} = 0$ , gives  $x = 0.68$ ,  $f(0.68) = -0.71$   $(0.68, -0.71)$  A1

c) for stationary points  $f'(x) = 3x^2 + 8e^{-2x}$  so that  $8e^{-2x} = -3x^2$  but  $8e^{-2x} > 0$  A1

this equation has no solutions, and therefore the graph of the function  $f$   
 has no stationary points. Colin is clearly incorrect. A1

