

**QCE
Specialist
Mathematics
Trial Examination
Paper 1: Tech-free
Section 1**



Kilbaha Education

Quality educational content

Kilbaha Education
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9018 5376
kilbaha@gmail.com
<https://kilbaha.com.au>

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Trial assessment 2021

Multiple choice question book

Specialist Mathematics

Paper 1— Technology-free

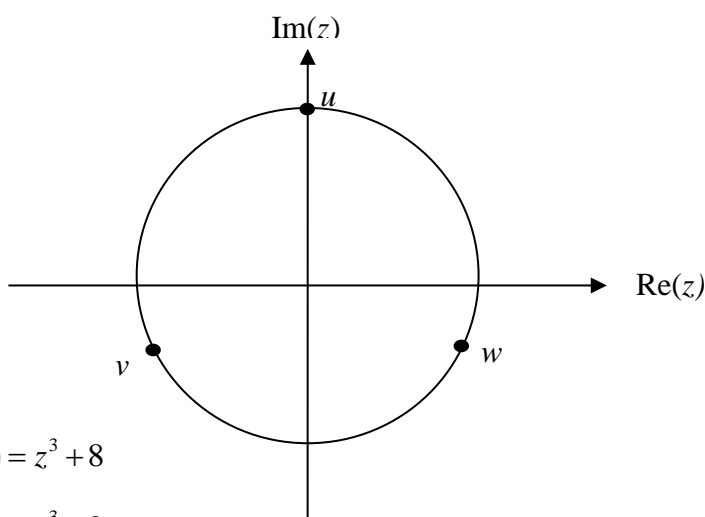
Section 1

Instructions

- Answer all questions in the question and response book.
- This book will not be marked.

QUESTION 1

The diagram shows a circle of radius 2 on an Argand diagram. The points shown u , v and w are equally spaced around the circle and are the solutions of the equation $P(z) = 0$. Then



- (A) $P(z) = z^3 + 8$
- (B) $P(z) = z^3 - 8$
- (C) $P(z) = z^3 + 8i$
- (D) $P(z) = z^3 - 8i$

QUESTION 2

Which one of the following relations is **not** the graph of a straight line passing through the origin with gradient -1 ?

- (A) $\{z: |z+1| = |z-i|\}$
- (B) $\{z: |z-1| = |z+i|\}$
- (C) $\{z: \operatorname{Re}(z) + \operatorname{Im}(z) = 0\}$
- (D) $\{z: \operatorname{Arg}(z) = -\frac{\pi}{4}\} \cup \{z: \operatorname{Arg}(z) = \frac{3\pi}{4}\}$

**QCE
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Section 2**



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kilbaha@gmail.com
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School name

Given name/s

Family name

Trial assessment 2021

Question and response book

Specialist Mathematics

Paper 1— Technology-free

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are not allowed.
- QCAA formula sheet provided.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (60 marks)

- 9 short response questions

Section 1

Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

	A	B	C	D
1.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Section 2

Instructions

- Write using black or blue pen.
 - Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
 - If you need more space for a response, use the additional pages at the back of this booklet.
 - On the additional pages, write the question number you are responding to.
 - Cancel any incorrect response by ruling a single diagonal line through your work.
 - Write the page number of your alternative/additional response, i.e. See page ...
 - If you do not do this, your original response will be marked.
 - This section has ten questions and is worth 60 marks.
-

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (6 marks)

Find an antiderivative of $\frac{3x-5}{\sqrt{25-9x^2}}$, for $x \in (-b, b)$, stating the value of b .

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$\frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = \pi r^2 h$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c$
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$
$\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\left(\cos^{-1}\left(\frac{x}{a}\right)\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

Calculus		
chain rule	If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
volume of a solid of revolution	about the x -axis	$V = \pi \int_a^b [f(x)]^2 dx$
	about the y -axis	$V = \pi \int_a^b [f(y)]^2 dy$
Simpson's rule	$\int_a^b f(x)dx \approx \frac{w}{3} [f(x_0) + 4[f(x_1) + f(x_3) + \dots] + 2[f(x_2) + f(x_4) + \dots] + f(x_n)]$	
simple harmonic motion	If $\frac{d^2x}{dt^2} = -\omega^2x$ then $x = A\sin(\omega t + \alpha)$ or $x = A\cos(\omega t + \beta)$	
	$v^2 = \omega^2(A^2 - x^2)$	$T = \frac{2\pi}{\omega}$
acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r \text{cis}(\theta)$
modulus	$ z = r = \sqrt{x^2 + y^2}$
argument	$\arg(z) = \theta, \tan(\theta) = \frac{y}{x}, -\pi < \theta \leq \pi$
product	$z_1 z_2 = r_1 r_2 \text{cis}(\theta_1 + \theta_2)$
quotient	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \text{cis}(\theta_1 - \theta_2)$
De Moivre's theorem	$z^n = r^n \text{cis}(n\theta)$

Statistics	
binomial theorem	$(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{r}x^{n-r}y^r + \dots + y^n$
permutations	${}^n P_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \dots \times (n-r+1)$
combinations	${}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
sample means	mean μ
	standard deviation $\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $\quad = 1 - 2\sin^2(A)$ $\quad = 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}(\cos(A - B) - \cos(A + B))$ $\cos(A)\cos(B) = \frac{1}{2}(\cos(A - B) + \cos(A + B))$ $\sin(A)\cos(B) = \frac{1}{2}(\sin(A + B) + \sin(A - B))$ $\cos(A)\sin(B) = \frac{1}{2}(\sin(A + B) - \sin(A - B))$

Vectors and matrices		
magnitude	$ \mathbf{a} = \left \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
scalar (dot) product	$\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \mathbf{b} \cos(\theta)$	
	$\mathbf{a} \cdot \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$	
vector equation of a line	$\mathbf{r} = \mathbf{a} + k \mathbf{d}$	
Cartesian equation of a line	$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$	
vector (cross) product	$\mathbf{a} \times \mathbf{b} = \mathbf{a} \mathbf{b} \sin(\theta) \hat{\mathbf{n}}$	
	$\mathbf{a} \times \mathbf{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
vector projection	\mathbf{a} on $\mathbf{b} = \mathbf{a} \cos(\theta) \hat{\mathbf{b}} = (\mathbf{a} \cdot \hat{\mathbf{b}}) \hat{\mathbf{b}}$	
vector equation of a plane	$\mathbf{r} \cdot \mathbf{n} = \mathbf{a} \cdot \mathbf{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(A) = ad - bc$	
multiplicative inverse matrices	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \det(A) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x \tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$
Physical constants		
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$	

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QUESTION 1 ANSWER C

The circle has radius 2, one of the solutions is $u = 2i$ and $u^3 = 8i^3 = -8i$
 but $P(u) = 0$ so $P(z) = z^3 + 8i = 0$

QUESTION 2 ANSWER D

Let $z = x + yi$ $\bar{z} = x - yi$, checking each alternative,

(A) $|z+1| = |z-i| \Rightarrow \sqrt{(x+1)^2 + y^2} = \sqrt{x^2 + (y-1)^2}$
 $x^2 + 2x + 1 + y^2 = x^2 + y^2 - 2y + 1 \Rightarrow y = -x$

(B) $|z-1| = |z+i| \Rightarrow \sqrt{(x-1)^2 + y^2} = \sqrt{x^2 + (y+1)^2}$
 $x^2 - 2x + 1 + y^2 = x^2 + y^2 + 2y + 1 \Rightarrow y = -x$

(C) $\text{Re}(z) + \text{Im}(z) = 0 \Rightarrow y = -x$

(D) $\{z : \text{Arg}(z) = -\frac{\pi}{4}\} \cup \{z : \text{Arg}(z) = \frac{3\pi}{4}\}$ are two rays from the origin, making angles
 of $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$ however the origin is **not** included, it is not the full line $y = -x$

QUESTION 3 ANSWER D

The vector $\underline{v} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ is perpendicular to the plane $2x - 3y + 4z = 12$.

QUESTION 4 ANSWER A

An approximate confidence interval for μ is given by $\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}}\right)$.

width of a confidence interval with sample size n is $\frac{2zs}{\sqrt{n}}$

width of a confidence interval with sample size $4n$ is $\frac{2zs}{\sqrt{4n}} = \frac{2zs}{2\sqrt{n}} = \frac{zs}{\sqrt{n}}$

The width of the confidence interval has been halved, that is it has been decreased by a factor of 2.

QUESTION 5 ANSWER C

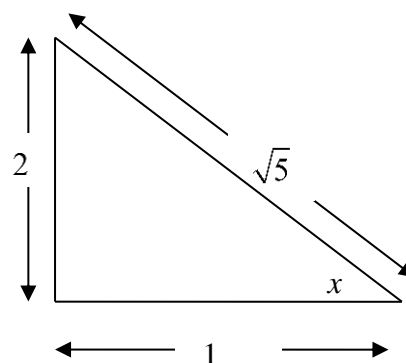
$\text{cosec}(x) = \frac{1}{\sin(x)} = \frac{\sqrt{5}}{2}$, $\sin(x) = \frac{2}{\sqrt{5}}$

Since $\frac{\pi}{2} < x < \pi$ x is in the 2nd quadrant

$\tan(x) < 0 \Rightarrow \tan(x) = -2$

$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)} = \frac{-4}{1 - 4} = \frac{4}{3}$

$\cot(2x) = \frac{3}{4}$



QUESTION 11

$$\int \frac{3x-5}{\sqrt{25-9x^2}} dx \quad \text{separate out into two integrals}$$

$$= 3 \int \frac{x}{\sqrt{25-9x^2}} dx - 5 \int \frac{1}{\sqrt{25-9x^2}} dx \quad \text{M1}$$

$$\text{let } u = 25 - 9x^2 \quad \text{let } v = 3x \quad \text{M1}$$

$$\frac{du}{dx} = -18x \quad \frac{dv}{dx} = 3$$

$$= -\frac{3}{18} \int u^{-\frac{1}{2}} du - \frac{5}{3} \int \frac{1}{\sqrt{25-v^2}} dv \quad \text{A2}$$

$$= -\frac{1}{6} \left(2u^{\frac{1}{2}} \right) + \frac{5}{3} \cos^{-1} \left(\frac{v}{5} \right) + c$$

$$= -\frac{1}{3} \sqrt{25-9x^2} + \frac{5}{3} \cos^{-1} \left(\frac{3x}{5} \right) + c \quad \text{A1}$$

$$\text{for } |x| < \frac{5}{3} \quad b = \frac{5}{3} \quad \text{A1}$$

$$\text{alternatively } = -\frac{1}{3} \sqrt{25-9x^2} - \frac{5}{3} \sin^{-1} \left(\frac{3x}{5} \right) + c \quad \text{for } |x| < \frac{5}{3} \quad b = \frac{5}{3}$$

QUESTION 12

$$v = \frac{dx}{dt} = \frac{9}{81-4t^2}$$

$$D = \int_0^3 \frac{9}{81-4t^2} dt \quad \text{by partial fractions} \quad \text{A1}$$

$$\frac{9}{81-4t^2} = \frac{A}{9-2t} + \frac{B}{9+2t} = \frac{A(9+2t) + B(9-2t)}{(9-2t)(9+2t)} = \frac{9(A+B) + 2t(A-B)}{81-4t^2}$$

$$A+B=1 \quad \text{M2}$$

$$A-B=0 \quad A=B=\frac{1}{2}$$

$$D = \frac{1}{2} \int_0^3 \left(\frac{1}{9+2t} + \frac{1}{9-2t} \right) dt \quad \text{A1}$$

$$D = \frac{1}{2} \left[\frac{1}{2} \log_e (|9+2t|) - \frac{1}{2} \log_e (|9-2t|) \right]_0^3 \quad \text{A1}$$

$$D = \frac{1}{4} \left[\log_e \left(\frac{9+2t}{9-2t} \right) \right]_0^3 = \frac{1}{4} \left[\log_e \left(\frac{15}{3} \right) - \log_e (1) \right]$$

$$D = \frac{1}{4} \log_e (5) \quad \text{A1}$$