QCE
Specialist Mathematics
Trial Examination
Paper 1:Tech-free
Section 1

Kilbaha Education

Quality educational content

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Trial assessment 2021

Multiple choice question book

Specialist Mathematics

Paper 1— Technology-free

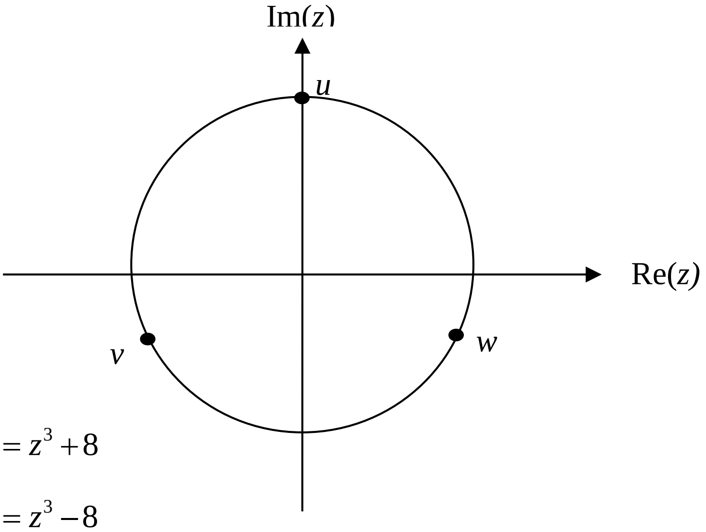
Section 1

Instructions

- Answer all questions in the question and response book.
- This book will not be marked.

QUESTION 1

The diagram shows a circle of radius 2 on an Argand diagram. The points shown u, v and w are equally spaced around the circle and are the solutions of the equation $P(z)=0$. Then



(A) $P(z)=z^3+8$

(B) $P(z)=z^3-8$

(C) $P(z)=z^3+8i$

(D) $P(z)=z^3-8i$

QUESTION 2

Which one of the following relations is **not** the graph of a straight line passing through the origin with gradient -1?

(A) $\{z:|z+1|=|z-i|\}$

(B) $\{z:|z-1|=|z+i|\}$

(C) $\{z:\operatorname{Re}(z)+\operatorname{Im}(z)=0\}$

(D) $\{z:\operatorname{Arg}(z)=-\frac{\pi}{4}\}\cup\{z:\operatorname{Arg}(z)=\frac{3\pi}{4}\}$

QCE Specialist Mathematics Trial Examination Paper 1:Tech-free Section 2

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School name

Given name/s

Family name

Trial assessment 2021

Question and response book

Specialist Mathematics

Paper 1— Technology-free

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this question and response booklet.
- Calculators are not allowed.
- QCAA formula sheet provided.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (60 marks)

- 9 short response questions

Section 1

Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	⬬	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.

– On the additional pages, write the question number you are responding to.
– Cancel any incorrect response by ruling a single diagonal line through your work.
– Write the page number of your alternative/additional response, i.e. See page …
– If you do not do this, your original response will be marked.

- This section has ten questions and is worth 60 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (6 marks)

Find an antiderivative of $\frac{3x-5}{\sqrt{25-9x^2}}$, for $x \in (-b, b)$, stating the value of b.

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Mensuration			
circumference of a circle	$C=2\pi r$	**area of a circle**	$A=\pi r^2$
area of a parallelogram	$A=bh$	**area of a trapezium**	$\frac{1}{2}(a+b)h$
area of a triangle	$A=\frac{1}{2}bh$	**total surface area of a cone**	$S=\pi rs+\pi r^2$
total surface area of a cylinder	$S=2\pi rh+2\pi r^2$	**surface area of a sphere**	$S=\pi r^2 h$
volume of a cone	$V=\frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V=\pi r^2 h$
volume of a prism	$V=Ah$	**volume of a pyramid**	$V=\frac{1}{3}Ah$
volume of a sphere	$V=\frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$	$\int x^n dx=\frac{1}{n+1}x^{n+1}+c$
$\frac{d}{dx}\left(e^x\right)=e^x$	$\int e^x dx=e^x+c$
$\frac{d}{dx}\left(\log_e(x)\right)=\frac{1}{x}$	$\int \frac{1}{x}dx=\log_e\left(\lvert x\rvert\right)+c$
$\frac{d}{dx}\left(\sin(x)\right)=\cos(x)$	$\int \sin(x)dx=-\cos(x)+c$
$\frac{d}{dx}\left(\cos(x)\right)=-\sin(x)$	$\int \cos(x)dx=\sin(x)+c$
$\frac{d}{dx}\left(\tan(x)\right)=\sec^2(x)$	$\int \sec^2(x)dx=\tan(x)+c$
$\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right)=\frac{1}{\sqrt{a^2-x^2}}$	$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+c$
$\frac{d}{dx}\left(\cos^{-1}\left(\frac{x}{a}\right)\right)=\frac{-1}{\sqrt{a^2-x^2}}$	$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+c$
$\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right)=\frac{a}{a^2+x^2}$	$\int \frac{a}{a^2+x^2}dx=\tan^{-1}\left(\frac{x}{a}\right)+c$

Calculus		
chain rule	If $h(x)=f(g(x))$ then $h'(x)=f'(g(x))g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
volume of a solid of revolution	about the x-axis	$V=\pi\int_a^b[f(x)]^2\,dx$
	about the y-axis	$V=\pi\int_a^b[f(y)]^2\,dy$
Simpson's rule	$\int_a^b f(x)dx\approx\frac{w}{3}\left[f(x_0)+4[f(x_1)+f(x_3)+\ldots]+2[f(x_2)+f(x_4)+\ldots]+f(x_n)\right]$	
simple harmonic motion	If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin(\omega t+\alpha)$ or $x=A\cos(\omega t+\beta)$	
	$v^2=\omega^2(A^2-x^2)$ $T=\frac{2\pi}{\omega}$	$f=\frac{1}{T}$
acceleration	$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r(\cos(\theta)+i\sin(\theta))=r\,\text{cis}(\theta)$
modulus	$\lvert z\rvert=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\frac{y}{x}\ ,\ -\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\text{cis}(\theta_1+\theta_2)$
quotient	$\frac{z_1}{z_2}=\frac{r_1}{r_2}\,\text{cis}(\theta_1-\theta_2)$
De Moivre's theorem	$z^n=r^n\text{cis}(n\theta)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \ldots\ldots + \binom{n}{r} x^{n-r} y^r + \ldots\ldots + y^n$	
permutations	${}^nP_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots \times (n-r+1)$	
combinations	${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\frac{s}{\sqrt{n}}, \bar{x} + z\frac{s}{\sqrt{n}}\right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ $\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B))$ $\sin(A)\cos(B) = \frac{1}{2}(\sin(A+B) + \sin(A-B))$ $\cos(A)\sin(B) = \frac{1}{2}(\sin(A+B) - \sin(A-B))$

Vectors and matrices	
magnitude	$\lvert\boldsymbol{a}\rvert = \left\lvert\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\right\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$
scalar (dot) product	$\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\cos(\theta)$
	$\boldsymbol{a}.\boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}.\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$
Cartesian equation of a line	$\dfrac{x-a_1}{d_1} = \dfrac{y-a_2}{d_2} = \dfrac{z-a_3}{d_3}$
vector (cross) product	$\boldsymbol{a}\times\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\lvert\boldsymbol{b}\rvert\sin(\theta)\hat{\boldsymbol{n}}$
	$\boldsymbol{a}\times\boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}\times\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert\boldsymbol{a}\rvert\cos(\theta)\hat{\boldsymbol{b}} = (\boldsymbol{a}.\hat{\boldsymbol{b}})\hat{\boldsymbol{b}}$
vector equation of a plane	$\boldsymbol{r}.\boldsymbol{n} = \boldsymbol{a}.\boldsymbol{n}$
Cartesian equation of a plane	$ax + by + cz + d = 0$
determinant	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\boldsymbol{A}) = ad - bc$
multiplicative inverse matrices	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{\det(\boldsymbol{A})}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, $\det(\boldsymbol{A}) \neq 0$
linear transformations	dilation: $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation: $\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$): $\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constants	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8\ \text{m s}^{-2}$

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Suggested Solutions
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QUESTION 1 ANSWER C

The circle has radius 2, one of the solutions is $u=2i$ and $u^3=8i^3=-8i$
but $P(u)=0$ so $P(z)=z^3+8i=0$

QUESTION 2 ANSWER D

Let $z=x+yi$ $\bar{z}=x-yi$, checking each alternative,

(A) $|z+1|=|z-i|$ $\Rightarrow\sqrt{(x+1)^2+y^2}=\sqrt{x^2+(y-1)^2}$

$x^2+2x+1+y^2=x^2+y^2-2y+1$ $\Rightarrow y=-x$

(B) $|z-1|=|z+i|$ $\Rightarrow\sqrt{(x-1)^2+y^2}=\sqrt{x^2+(y+1)^2}$

$x^2-2x+1+y^2=x^2+y^2+2y+1$ $\Rightarrow y=-x$

(C) $\text{Re}(z)+\text{Im}(z)=0$ $\Rightarrow y=-x$

(D) $\{z:\text{Arg}(z)=-\frac{\pi}{4}\}\cup\{z:\text{Arg}(z)=\frac{3\pi}{4}\}$ are two rays from the origin, making angles of $-\frac{\pi}{4}$ and $\frac{3\pi}{4}$ however the origin is **not** included, it is not the full line $y=-x$

QUESTION 11

$\int \frac{3x-5}{\sqrt{25-9x^2}}\,dx$ separate out into two integrals

$= 3\int \frac{x}{\sqrt{25-9x^2}}\,dx - 5\int \frac{1}{\sqrt{25-9x^2}}\,dx$ M1

let $u = 25-9x^2$ let $v = 3x$

$\frac{du}{dx} = -18x$ $\frac{dv}{dx} = 3$ M1

$= -\frac{3}{18}\int u^{-\frac{1}{2}}\,du - \frac{5}{3}\int \frac{1}{\sqrt{25-v^2}}\,dv$

A2

$= -\frac{1}{6}\left(2u^{\frac{1}{2}}\right) + \frac{5}{3}\cos^{-1}\left(\frac{v}{5}\right) + c$

$= -\frac{1}{3}\sqrt{25-9x^2} + \frac{5}{3}\cos^{-1}\left(\frac{3x}{5}\right) + c$ A1

for $|x| < \frac{5}{3}$ $b = \frac{5}{3}$ A1

alternatively $= -\frac{1}{3}\sqrt{25-9x^2} - \frac{5}{3}\sin^{-1}\left(\frac{3x}{5}\right) + c$ for $|x| < \frac{5}{3}$ $b = \frac{5}{3}$