

**QCE
Specialist
Mathematics
Trial Examination
Paper 2: Tech-active
Section 1**



Kilbaha Education

Quality educational content

Kilbaha Education
PO Box 2227
Kew Vic 3101
Australia

Tel: (03) 9018 5376
kilbaha@gmail.com
<https://kilbaha.com.au>

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Trial assessment 2021

Multiple choice question book

Specialist Mathematics

Paper 2— Technology-active

Section 1

Instructions

- Answer all questions in the question and response book.
 - This book will not be marked.
-

QUESTION 1

Four players A,B,C and D competed in a table tennis round robin competition, where all players played each other once. The matrix below is the dominance matrix, where a 1 indicates defeated and a 0 represents was defeated by,

$$N = \begin{array}{c} \text{Losing} \\ \text{A B C D} \\ \text{Winning} \end{array} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

Consider the following statements

- 1) A won one match and defeated player C.
B won two matches and defeated players A and C.
C lost all matches.
D won all three matches.
- 2) The ranking order is D,B,A,C.
- 3) The ranking order is C,A,B,D.

Then

- (A) only statement 1 is true.
- (B) only statement 2 is true.
- (C) statements 1) and 2) are both true.
- (D) statements 1) and 3) are both true.

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Section 2**



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School name

Given name/s

Family name

Trial assessment 2021

Question and response book

Specialist Mathematics

Paper 2— Technology-active

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this questions and response book.
- QCAA approved calculator permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (60 marks)

- 10 short response questions

Section 1

Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	<input checked="" type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

	A	B	C	D
1.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
2.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
3.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
4.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
5.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
6.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
7.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
8.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
9.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>
10.	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>

Section 2

Instructions

- Write using black or blue pen.
 - Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
 - If you need more space for a response, use the additional pages at the back of this booklet.
 - On the additional pages, write the question number you are responding to.
 - Cancel any incorrect response by ruling a single diagonal line through your work.
 - Write the page number of your alternative/additional response, i.e. See page ...
 - If you do not do this, your original response will be marked.
 - This section has ten questions and is worth 60 marks.
-

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

Let $z = -1 - i$

a) Express z in polar form $r \operatorname{cis}(\theta)$, where $r, \theta \in \mathbb{R}$.

[1 mark]

b) Find z^7 giving your answer in both rectangular and polar form.

[2 marks]

c) Determine $\operatorname{Arg}(z^7)$.

[1 mark]

Mensuration			
circumference of a circle	$C = 2\pi r$	area of a circle	$A = \pi r^2$
area of a parallelogram	$A = bh$	area of a trapezium	$\frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	total surface area of a cone	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	surface area of a sphere	$S = \pi r^2 h$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	volume of a cylinder	$V = \pi r^2 h$
volume of a prism	$V = Ah$	volume of a pyramid	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}(x^n) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c$
$\frac{d}{dx}(e^x) = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}(\log_e(x)) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e(x) + c$
$\frac{d}{dx}(\sin(x)) = \cos(x)$	$\int \sin(x) dx = -\cos(x) + c$
$\frac{d}{dx}(\cos(x)) = -\sin(x)$	$\int \cos(x) dx = \sin(x) + c$
$\frac{d}{dx}(\tan(x)) = \sec^2(x)$	$\int \sec^2(x) dx = \tan(x) + c$
$\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\left(\cos^{-1}\left(\frac{x}{a}\right)\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

Calculus		
chain rule	If $h(x) = f(g(x))$ then $h'(x) = f'(g(x))g'(x)$	If $y = f(u)$ and $u = g(x)$ then $\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$
product rule	If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}$
quotient rule	If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$	$\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
volume of a solid of revolution	about the x -axis	$V = \pi \int_a^b [f(x)]^2 dx$
	about the y -axis	$V = \pi \int_a^b [f(y)]^2 dy$
Simpson's rule	$\int_a^b f(x)dx \approx \frac{w}{3} [f(x_0) + 4[f(x_1) + f(x_3) + \dots] + 2[f(x_2) + f(x_4) + \dots] + f(x_n)]$	
simple harmonic motion	If $\frac{d^2x}{dt^2} = -\omega^2x$ then $x = A\sin(\omega t + \alpha)$ or $x = A\cos(\omega t + \beta)$	
	$v^2 = \omega^2(A^2 - x^2)$	$T = \frac{2\pi}{\omega}$
acceleration	$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = v \frac{dv}{dx} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z = x + yi = r(\cos(\theta) + i\sin(\theta)) = r \operatorname{cis}(\theta)$
modulus	$ z = r = \sqrt{x^2 + y^2}$
argument	$\arg(z) = \theta, \tan(\theta) = \frac{y}{x}, -\pi < \theta \leq \pi$
product	$z_1 z_2 = r_1 r_2 \operatorname{cis}(\theta_1 + \theta_2)$
quotient	$\frac{z_1}{z_2} = \frac{r_1}{r_2} \operatorname{cis}(\theta_1 - \theta_2)$
De Moivre's theorem	$z^n = r^n \operatorname{cis}(n\theta)$

Statistics	
binomial theorem	$(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \dots + \binom{n}{r}x^{n-r}y^r + \dots + y^n$
permutations	${}^n P_r = \frac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \dots \times (n-r+1)$
combinations	${}^n C_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$
sample means	mean μ
	standard deviation $\frac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z \frac{s}{\sqrt{n}}, \bar{x} + z \frac{s}{\sqrt{n}} \right)$

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A - B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A - B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $\quad = 1 - 2\sin^2(A)$ $\quad = 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}(\cos(A - B) - \cos(A + B))$ $\cos(A)\cos(B) = \frac{1}{2}(\cos(A - B) + \cos(A + B))$ $\sin(A)\cos(B) = \frac{1}{2}(\sin(A + B) + \sin(A - B))$ $\cos(A)\sin(B) = \frac{1}{2}(\sin(A + B) - \sin(A - B))$

Vectors and matrices		
magnitude	$ a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
scalar (dot) product	$a \cdot b = a b \cos(\theta)$	
	$a \cdot b = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1b_1 + a_2b_2 + a_3b_3$	
vector equation of a line	$r = a + kd$	
Cartesian equation of a line	$\frac{x - a_1}{d_1} = \frac{y - a_2}{d_2} = \frac{z - a_3}{d_3}$	
vector (cross) product	$a \times b = a b \sin(\theta)\hat{n}$	
	$a \times b = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$	
vector projection	$a \text{ on } b = a \cos(\theta)\hat{b} = (a \cdot \hat{b})\hat{b}$	
vector equation of a plane	$r \cdot n = a \cdot n$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(A) = ad - bc$	
multiplicative inverse matrices	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{\det(A)} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}, \det(A) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x \tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$
Physical constants		
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ ms}^{-2}$	

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QUESTION 1 ANSWER C

A won one match and defeated player C.

B won two matches and defeated players A and C.

C lost all matches.

D won all three matches.

The ranking order is D,B,A,C.

QUESTION 2 ANSWER D

as the value of n gets larger and larger $\begin{bmatrix} 0.2 & 0.8 & 0.3 \\ 0 & 0.4 & 0 \\ 0.6 & 0 & 0 \end{bmatrix}^n \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

QUESTION 3 ANSWER B

$P(z)$ is a fourth degree polynomial

$P(z) = (z - ki)(z + ki)(z - 2k)(z + k)$ expanding

$$P(z) = (z^2 + k^2)(z^2 - kz - 2k^2)$$

$$P(z) = z^4 - kz^3 - k^2z^2 - k^3z - 2k^4$$

QUESTION 4 ANSWER D

$$\frac{2 - ki}{3 + ki} = \frac{2 - ki}{3 + ki} \times \frac{3 - ki}{3 - ki} = \frac{6 - k^2 - 5ki}{9 + k^2}$$

$$\operatorname{Re}\left(\frac{6 - k^2 - 5ki}{9 + k^2}\right) = 0 \Rightarrow 6 - k^2 = 0$$

$$k = \pm\sqrt{6} \text{ only.}$$

QUESTION 5 ANSWER A

$$\{z: |z - a|^2 - |z - bi|^2 = a^2 + b^2\} \text{ let } z = x + yi$$

$$|(x - a + yi)|^2 - |x + (y - b)i|^2 = a^2 + b^2$$

$$(x - a)^2 + y^2 - (x^2 + (y - b)^2) = a^2 + b^2$$

$$x^2 - 2xa + a^2 + y^2 - (x^2 + y^2 - 2by + b^2) = a^2 + b^2$$

$$2by - 2xa = 2b^2$$

$$y = \frac{xa}{b} + b \text{ or } z = x + yi \quad \operatorname{Im}(z) = \frac{a}{b} \operatorname{Re}(z) + b$$

this represents a straight line in the Argand plane.

QUESTION 11

a) $z = -1 - i = \sqrt{2} \operatorname{cis}\left(-\frac{3\pi}{4}\right)$ A1

b) $z^7 = \left(\sqrt{2} \operatorname{cis}\left(-\frac{3\pi}{4}\right)\right)^7 = (\sqrt{2})^7 \operatorname{cis}\left(-\frac{21\pi}{4}\right)$
 $z^7 = 8\sqrt{2} \operatorname{cis}\left(-\frac{21\pi}{4} + 6\pi\right) = 8\sqrt{2} \operatorname{cis}\left(\frac{3\pi}{4}\right)$ M1
 $z^7 = 8\sqrt{2} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right)\right) = 8\sqrt{2} \left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$
 $z^7 = (-1 - i)^7 = -8 + 8i$ A1

c) $\operatorname{Arg}\left((-1 - i)^7\right) = \frac{3\pi}{4}$ A1

QUESTION 12

a) $v^2 = 63 - 54x - 9x^2$
 $\frac{d}{dx}\left(\frac{1}{2}v^2\right) = \frac{1}{2} \frac{d}{dx}(63 - 54x - 9x^2) = \frac{1}{2}(-54 - 18x)$ A1
 $\ddot{x} = -9(x + 3)$

This is of the form $\ddot{x} = -n^2(x - c)$ which proves the motion is simple harmonic.

The period is $T = \frac{2\pi}{n}$ is $\frac{2\pi}{3}$ s A1

now $v = 63 - 54x - 9x^2 = -9(x^2 + 6x - 7) = -9(x + 7)(x - 1)$, when $v = 0$, $x = -7, 1$
 so the particle moves between $x = -7$ and $x = 1$, so its amplitude is 4.

the centre of the motion is at $x = c = -3$. A1

b) since the particle starts at its endpoint, $x(1) = 0$, $x(t) = -3 + 4 \cos(3t)$ A2
 or $x(t) = -3 + 4 \sin\left(3t + \frac{\pi}{2}\right)$ or $x(t) = -3 + 4 \sin\left(\frac{\pi}{2} - 3t\right)$

QUESTION 13

a) $m = 1.2$ tonnes, $m = 1200$ kg, $r = 50$ m, $v = 60$ km/hr $v = 16.6$ m/s
 $N = mg - \frac{mv^2}{r} = m\left(g - \frac{v^2}{r}\right) = 1200\left(9.8 - \frac{16.6^2}{50}\right)$ M1
 $N = 5093.33$ Newtons A1

b) $v = ?$ $N = 0$
 $mg = \frac{mv^2}{r}$ M1
 $v = \sqrt{gr} = \sqrt{9.8 \times 50} = 22.135$ m/s $= 22.135 \times \frac{60 \times 60}{1000}$ A1
 $v = 79.69$ km/hr