QCE
Specialist Mathematics
Trial Examination
Paper 2:Tech-active
Section 1

Kilbaha Education

Quality educational content

Kilbaha Education **PO Box 2227** **Kew Vic 3101** **Australia**	**Tel: (03) 9018 5376** kilbaha@gmail.com https://kilbaha.com.au

Trial assessment 2021

Multiple choice question book

Specialist Mathematics

Paper 2— Technology-active

Section 1

Instructions

- Answer all questions in the question and response book.
- This book will not be marked.

QUESTION 1

Four players A,B,C and D competed in a table tennis round robin competition, where all players played each other once. The matrix below is the dominance matrix, where a 1 indicates defeated and a 0 represents was defeated by,

$$N = \text{Winning} \begin{array}{c} \text{Losing} \\ \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Consider the following statements

1) A won one match and defeated player C.
   B won two matches and defeated players A and C.
   C lost all matches.
   D won all three matches.
2) The ranking order is D,B,A,C.
3) The ranking order is C,A,B,D.

Then

(A) only statement 1 is true.

(B) only statement 2 is true.

(C) statements 1) and 2) are both true.

(D) statements 1) and 3) are both true.

QCE Specialist Mathematics Trial Examination Paper 2:Tech-active Section 2

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School name

Given name/s

Family name

Trial assessment 2021

Question and response book

Specialist Mathematics

Paper 2— Technology-active

Time allowed

- Perusal time – 5 minutes
- Working time – 90 minutes

General instructions

- Answer all questions in this questions and response book.
- QCAA approved calculator permitted.
- QCAA formula sheet provided.
- Planning paper will not be marked.

Section 1 (10 marks)

- 10 multiple choice questions

Section 2 (60 marks)

- 10 short response questions

Section 1

Instructions

- Chose the best answer for Questions 1-10.
- This section has 10 questions and is worth 10 marks.
- Use a 2B pencil in the A, B, C, or D answer bubble completely.
- If you change your mind or make a mistake, use an eraser to remove your response and fill in the new answer bubble completely.

	A	B	C	D
Example:	●	○	○	○

	A	B	C	D
1.	○	○	○	○
2.	○	○	○	○
3.	○	○	○	○
4.	○	○	○	○
5.	○	○	○	○
6.	○	○	○	○
7.	○	○	○	○
8.	○	○	○	○
9.	○	○	○	○
10.	○	○	○	○

Section 2

Instructions

- Write using black or blue pen.
- Questions worth more than one mark require mathematical reasoning and/or working to be shown to support answers.
- If you need more space for a response, use the additional pages at the back of this booklet.

– On the additional pages, write the question number you are responding to.
– Cancel any incorrect response by ruling a single diagonal line through your work.
– Write the page number of your alternative/additional response, i.e. See page …
– If you do not do this, your original response will be marked.

- This section has ten questions and is worth 60 marks.

DO NOT WRITE ON THIS PAGE

THIS PAGE WILL NOT BE MARKED

QUESTION 11 (4 marks)

Let $z = -1 - i$

a) Express z in polar form $r\,\text{cis}(\theta)$, where $r, \theta \in R$.

[1 mark]

b) Find z^7 giving your answer in both rectangular and polar form.

[2 marks]

c) Determine $\text{Arg}(z^7)$.

[1 mark]

ADDITIONAL PAGE FOR STUDENT RESPONSES

Write the question number you are responding to.

Mensuration			
circumference of a circle	$C = 2\pi r$	**area of a circle**	$A = \pi r^2$
area of a parallelogram	$A = bh$	**area of a trapezium**	$\frac{1}{2}(a+b)h$
area of a triangle	$A = \frac{1}{2}bh$	**total surface area of a cone**	$S = \pi rs + \pi r^2$
total surface area of a cylinder	$S = 2\pi rh + 2\pi r^2$	**surface area of a sphere**	$S = \pi r^2 h$
volume of a cone	$V = \frac{1}{3}\pi r^2 h$	**volume of a cylinder**	$V = \pi r^2 h$
volume of a prism	$V = Ah$	**volume of a pyramid**	$V = \frac{1}{3}Ah$
volume of a sphere	$V = \frac{4}{3}\pi r^3$		

Calculus	
$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$	$\int x^n dx = \frac{1}{n+1}x^{n+1} + c$
$\frac{d}{dx}\left(e^x\right) = e^x$	$\int e^x dx = e^x + c$
$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$	$\int \frac{1}{x} dx = \log_e\left(\lvert x \rvert\right) + c$
$\frac{d}{dx}\left(\sin(x)\right) = \cos(x)$	$\int \sin(x)\,dx = -\cos(x) + c$
$\frac{d}{dx}\left(\cos(x)\right) = -\sin(x)$	$\int \cos(x)\,dx = \sin(x) + c$
$\frac{d}{dx}\left(\tan(x)\right) = \sec^2(x)$	$\int \sec^2(x)\,dx = \tan(x) + c$
$\frac{d}{dx}\left(\sin^{-1}\left(\frac{x}{a}\right)\right) = \frac{1}{\sqrt{a^2 - x^2}}$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\left(\cos^{-1}\left(\frac{x}{a}\right)\right) = \frac{-1}{\sqrt{a^2 - x^2}}$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + c$
$\frac{d}{dx}\left(\tan^{-1}\left(\frac{x}{a}\right)\right) = \frac{a}{a^2 + x^2}$	$\int \frac{a}{a^2 + x^2} dx = \tan^{-1}\left(\frac{x}{a}\right) + c$

Calculus		
chain rule	If $h(x)=f(g(x))$ then $h'(x)=f'(g(x))g'(x)$	If $y=f(u)$ and $u=g(x)$ then $\frac{dy}{dx}=\frac{dy}{du}\frac{du}{dx}$
product rule	If $h(x)=f(x)g(x)$ then $h'(x)=f(x)g'(x)+f'(x)g(x)$	$\frac{d}{dx}(uv)=u\frac{dv}{dx}+v\frac{du}{dx}$
quotient rule	If $h(x)=\frac{f(x)}{g(x)}$ then $h'(x)=\frac{f'(x)g(x)-f(x)g'(x)}{(g(x))^2}$	$\frac{d}{dx}\left(\frac{u}{v}\right)=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$
integration by parts	$\int f(x)g'(x)dx=f(x)g(x)-\int f'(x)g(x)dx$	$\int u\frac{dv}{dx}dx=uv-\int v\frac{du}{dx}dx$
volume of a solid of revolution	about the *x*-axis	$V=\pi\int_a^b[f(x)]^2dx$
	about the *y*-axis	$V=\pi\int_a^b[f(y)]^2dy$
Simpson's rule	$\int_a^b f(x)dx\approx\frac{w}{3}\left[f(x_0)+4[f(x_1)+f(x_3)+...]+2[f(x_2)+f(x_4)+...]+f(x_n)\right]$	
simple harmonic motion	If $\frac{d^2x}{dt^2}=-\omega^2x$ then $x=A\sin(\omega t+\alpha)$ or $x=A\cos(\omega t+\beta)$	
	$v^2=\omega^2(A^2-x^2)$; $T=\frac{2\pi}{\omega}$	$f=\frac{1}{T}$
acceleration	$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=v\frac{dv}{dx}=\frac{d}{dx}\left(\frac{1}{2}v^2\right)$	

Real and complex numbers	
complex number forms	$z=x+yi=r(\cos(\theta)+i\sin(\theta))=r\,\text{cis}(\theta)$
modulus	$\|z\|=r=\sqrt{x^2+y^2}$
argument	$\arg(z)=\theta,\ \tan(\theta)=\frac{y}{x}\ ,\ -\pi<\theta\le\pi$
product	$z_1z_2=r_1r_2\,\text{cis}(\theta_1+\theta_2)$
quotient	$\frac{z_1}{z_2}=\frac{r_1}{r_2}\text{cis}(\theta_1-\theta_2)$
De Moivre's theorem	$z^n=r^n\text{cis}(n\theta)$

Statistics		
binomial theorem	$(x+y)^n = x^n + \binom{n}{1} x^{n-1} y + \ldots\ldots + \binom{n}{r} x^{n-r} y^r + \ldots\ldots + y^n$	
permutations	${}^nP_r = \dfrac{n!}{(n-r)!} = n \times (n-1) \times (n-2) \times \ldots\ldots \times (n-r+1)$	
combinations	${}^nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$	
sample means	mean	μ
	standard deviation	$\dfrac{\sigma}{\sqrt{n}}$
approximate confidence interval for μ	$\left(\bar{x} - z\dfrac{s}{\sqrt{n}},\ \bar{x} + z\dfrac{s}{\sqrt{n}} \right)$	

Trigonometry	
Pythagorean identities	$\sin^2(A) + \cos^2(A) = 1$ $\tan^2(A) + 1 = \sec^2(A)$ $\cot^2(A) + 1 = \operatorname{cosec}^2(A)$
angle sum and difference identities	$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$ $\sin(A-B) = \sin(A)\cos(B) - \cos(A)\sin(B)$ $\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$ $\cos(A-B) = \cos(A)\cos(B) + \sin(A)\sin(B)$
double-angle identities	$\sin(2A) = 2\sin(A)\cos(A)$ $\cos(2A) = \cos^2(A) - \sin^2(A)$ $= 1 - 2\sin^2(A)$ $= 2\cos^2(A) - 1$
product identities	$\sin(A)\sin(B) = \frac{1}{2}(\cos(A-B) - \cos(A+B))$ $\cos(A)\cos(B) = \frac{1}{2}(\cos(A-B) + \cos(A+B))$ $\sin(A)\cos(B) = \frac{1}{2}(\sin(A+B) + \sin(A-B))$ $\cos(A)\sin(B) = \frac{1}{2}(\sin(A+B) - \sin(A-B))$

Vectors and matrices		
magnitude	$\lvert\boldsymbol{a}\rvert = \left\lvert \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \right\rvert = \sqrt{a_1^2 + a_2^2 + a_3^2}$	
scalar (dot) product	$\boldsymbol{a}.\boldsymbol{b} = \lvert\boldsymbol{a}\rvert \lvert\boldsymbol{b}\rvert \cos(\theta)$	
	$\boldsymbol{a}.\boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$	
vector equation of a line	$\boldsymbol{r} = \boldsymbol{a} + k\boldsymbol{d}$	
Cartesian equation of a line	$\dfrac{x - a_1}{d_1} = \dfrac{y - a_2}{d_2} = \dfrac{z - a_3}{d_3}$	
vector (cross) product	$\boldsymbol{a} \times \boldsymbol{b} = \lvert\boldsymbol{a}\rvert \lvert\boldsymbol{b}\rvert \sin(\theta)\hat{\boldsymbol{n}}$	
	$\boldsymbol{a} \times \boldsymbol{b} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2 b_3 - a_3 b_2 \\ a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{pmatrix}$	
vector projection	$\boldsymbol{a}$ on $\boldsymbol{b} = \lvert\boldsymbol{a}\rvert \cos(\theta)\hat{\boldsymbol{b}} = (\boldsymbol{a}.\hat{\boldsymbol{b}})\hat{\boldsymbol{b}}$	
vector equation of a plane	$\boldsymbol{r}.\boldsymbol{n} = \boldsymbol{a}.\boldsymbol{n}$	
Cartesian equation of a plane	$ax + by + cz + d = 0$	
determinant	If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det(\boldsymbol{A}) = ad - bc$	
multiplicative inverse matrices	$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \dfrac{1}{\det(\boldsymbol{A})} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, $\det(\boldsymbol{A}) \neq 0$	
linear transformations	dilation	$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$
	rotation	$\begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}$
	reflection (in the line $y = x\tan(\theta)$)	$\begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix}$

Physical constants	
magnitude of mean acceleration due to gravity on Earth	$g = 9.8 \text{ m s}^{-2}$

QCE Specialist Mathematics Suggested Solutions Trial Paper 2 Tech-active

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QUESTION 1 ANSWER C

A won one match and defeated player C.

B won two matches and defeated players A and C.

C lost all matches.

D won all three matches.

The ranking order is D,B,A,C.

QUESTION 2 ANSWER D

as the value of n gets larger and larger $\begin{bmatrix} 0.2 & 0.8 & 0.3 \\ 0 & 0.4 & 0 \\ 0.6 & 0 & 0 \end{bmatrix}^n \begin{bmatrix} 10 \\ 20 \\ 30 \end{bmatrix} \to \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

QUESTION 11

a) $z = -1 - i = \sqrt{2}\,\text{cis}\left(-\frac{3\pi}{4}\right)$ A1

b) $z^7 = \left(\sqrt{2}\,\text{cis}\left(-\frac{3\pi}{4}\right)\right)^7 = \left(\sqrt{2}\right)^7 \text{cis}\left(-\frac{21\pi}{4}\right)$

$z^7 = 8\sqrt{2}\text{cis}\left(-\frac{21\pi}{4} + 6\pi\right) = 8\sqrt{2}\text{cis}\left(\frac{3\pi}{4}\right)$

M1

$z^7 = 8\sqrt{2}\left(\cos\left(\frac{3\pi}{4}\right) + i\sin\left(\frac{3\pi}{4}\right)\right) = 8\sqrt{2}\left(-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i\right)$

$z^7 = \left(-1-i\right)^7 = -8 + 8i$ A1

c) $\text{Arg}\left(\left(-1-i\right)^7\right) = \frac{3\pi}{4}$ A1