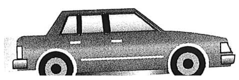
Question 23

A car of mass *m* kg is travelling on a level roadway. The engine exerts a constant propulsive force of *F* newtons and the total resistance to the motion of the car is kv newtons, where *k* is positive constant and *v* is its speed in m/s. The car moves from rest, and travels a distance of *D* metres until it obtains a speed of *V* m/s, in a time of *T* seconds.



Five students stated some relationships between the constants, *m*, *V*, *k*, *F* , *D* and *T*.

Alan stated that $mV = (F - kV)T$

Ben stated that $2mD = (F - kV)T^2$

Colin stated that $\frac{1}{2}mV^2 = (F - kV)D$

David stated that $D = \int_0^V \frac{mv}{F - kv}\,dv$

Edward stated that $T = \int_0^V \frac{m}{F - kv}\,dv$

Then

A. Alan, Ben and Colin are all correct.

B. Alan and Colin are both correct.

C. Only Colin is correct.

D. David and Edward are both correct.

E. Only Edward is correct.

ANSWER D

Resolving the forces horizontally, the equation of motion is $ma = F - kv,$ although, *m* , *V*, *k* , *F* , *D* and *T* are all constants, we cannot use the constant acceleration formulae, since the acceleration is not constant, and depends on the speed. So Alan, Ben and Colin are all incorrect, all their relationships are derived from using the constant acceleration formulae.

Use $a = \frac{dv}{dt}$ then $m\frac{dv}{dt} = F - kv \Rightarrow \frac{dt}{dv} = \frac{m}{F - kv}$ integrating, between $v = 0$ and $v = V$

$\Rightarrow \int_0^T 1\,dt = T = \int_0^V \frac{m}{F - kv}\,dv$, so that Edward is correct.

Use $a = v\frac{dv}{dx}$ then $mv\frac{dv}{dx} = F - kv \Rightarrow \frac{dx}{dv} = \frac{mv}{F - kv}$ integrating, between $v = 0$ and $v = V$,

$\Rightarrow \int_0^D 1\,dx = D = \int_0^V \frac{mv}{F - kv}\,dv$, so that David is correct.