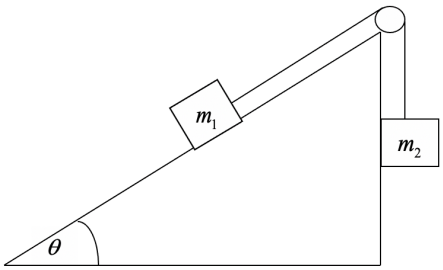
Question 11

A particle of mass m_1 kg is on a smooth plane, inclined at an angle of θ to the horizontal. It is connected by a light string which passes around a smooth pulley to another mass of m_2 kg hanging vertically, as shown in the diagram.



Then which of the following is **false?**

A. If $\theta = 30^0$ and $\frac{m_2}{m_1} = \frac{1}{2}$ then the system is in equilibrium.

B. If $\theta = 30^0$ and $\frac{m_2}{m_1} < \frac{1}{2}$ then the mass m_2 moves upwards.

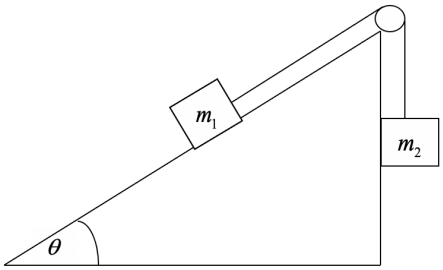
C. If $\theta = 45^0$ and $\frac{m_2}{m_1} = \frac{\sqrt{2}}{2}$ then the system is in equilibrium.

D. If $\theta = 60^0$ and $\frac{m_2}{m_1} = \frac{\sqrt{3}}{2}$ then the system is in equilibrium.

E. If $\theta = 60^0$ and $\frac{m_2}{m_1} < \frac{\sqrt{3}}{2}$ then the mass m_2 moves downwards.

ANSWER E

See next page . . .



resolving up parallel to plane around the m_1 kg mass $(1)\ \ T - m_1 g\sin(\theta) = m_1 a$

resolving downwards around the m_2 kg mass $(2)\ \ m_2 g - T = m_2 a$

adding to eliminate the tension in the string, to find the acceleration *a*, of the system

$(1)+(2)\ \ m_2 g - m_1 g\sin(\theta) = m_1 a + m_2 a \ \Rightarrow\ a = \dfrac{g\left(m_2 - m_1\sin(\theta)\right)}{m_1 + m_2}$

Checking the alternatives

$a > 0$ when $m_2 > m_1\sin(\theta) \ \Rightarrow\ \dfrac{m_2}{m_1} > \sin(\theta)$

and $a = 0$ when $\dfrac{m_2}{m_1} = \sin(\theta)$

If $\theta = 30^0$ and $\dfrac{m_2}{m_1} = \sin\left(30^0\right) = \dfrac{1}{2}$ then the system is in equilibrium, **A.** is correct

If $\theta = 30^0$ $a = \dfrac{g\left(m_2 - \dfrac{m_1}{2}\right)}{m_1 + m_2}$ if $\dfrac{m_2}{m_1} < \dfrac{1}{2}$ then $a < 0$

therefore the mass m_2 moves upwards, **B.** is correct

If $\theta = 45^0$ and $\dfrac{m_2}{m_1} = \sin\left(45^0\right) = \dfrac{\sqrt{2}}{2}$ then the system is in equilibrium, **C.** is correct

If $\theta = 60^0$ and $\dfrac{m_2}{m_1} = \sin\left(60^0\right) = \dfrac{\sqrt{3}}{2}$ then the system is in equilibrium, **D.** is correct

If $\theta = 60^0$ $a = \dfrac{g\left(m_2 - \dfrac{\sqrt{3}\,m_1}{2}\right)}{m_1 + m_2}$ if $\dfrac{m_2}{m_1} < \dfrac{\sqrt{3}}{2}$ then $a < 0$

therefore the mass m_2 moves upwards, **E.** is incorrect

